

# Thermodynamic Assessment of the Mn-B System

Weihua Sun, Yong Du, Shuhong Liu, Baiyun Huang, and Chao Jiang

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The phase equilibrium and thermodynamic data on the Mn-B system are critically reviewed. Based on the experimental data, a thermodynamic modeling is performed on this system. A set of self-consistent thermodynamic parameters is obtained and the calculated results are in general agreement with the experimental data. A parameter  $\xi$  is introduced to compare the degree of flatness of the  $\delta\text{Mn}/\text{L}$  liquidus with those in other metal-boron systems. It is found that the  $\delta\text{Mn}/\text{L}$  liquidus is the flattest, which causes difficulties in its modeling. An effective approach to evaluate the thermodynamic parameters for  $\alpha\text{-Mn}_2\text{B}_{0.982}$  phase is developed. Such an approach is valid for evaluating thermodynamic parameters of the phases with limited phase diagram data.

**Keywords** Mn-B system, phase diagram, thermodynamic modeling

## 1. Introduction

Boron and manganese are useful alloying elements that can improve the strength of many metal (e.g., Al, Cu, Fe, and Ni) based materials. In order to construct a thermodynamic database for those materials, a thermodynamic assessment on the Mn-B subsystem is necessary. The Mn-B system has been assessed by four groups of researchers.<sup>[1–4]</sup> Among them, Liao and Spear<sup>[3]</sup> have made a critical literature review and thermodynamic modeling on this system. However, a thermodynamic reassessment of the Mn-B system is still meaningful for the following reasons. First, more accurate lattice stabilities<sup>[5]</sup> for the pure elements are available. Second, when using the parameters from Liao and Spear,<sup>[3]</sup> the calculated phase diagram shows two miscibility gaps associated with liquid phase, as shown in Fig. 1. Finally, new partial enthalpy of mixing for the liquid phase has been measured by Vitusevich.<sup>[6]</sup> The purpose of this study is to critically review the literature data and provide a set of self-consistent thermodynamic parameters for the Mn-B system by means of the CALculation of PHAse Diagrams (CALPHAD) approach.

## 2. Literature Review

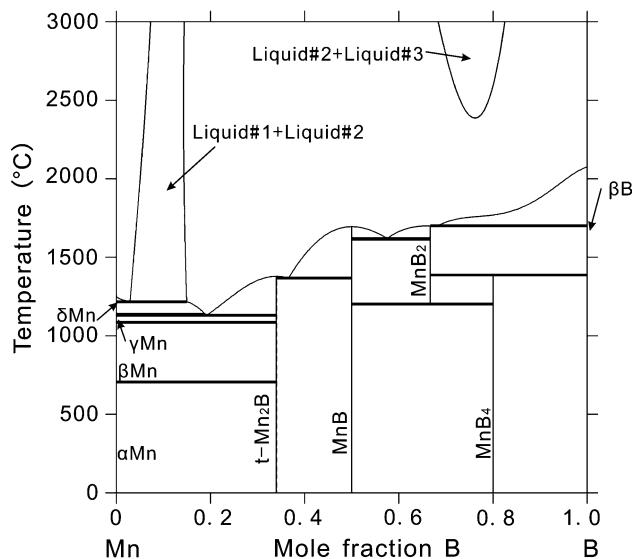
### 2.1 Phase Diagram Data

In order to facilitate reading, the crystal structure data on all the phases reported in the literature are listed in Table 1. By using a combination of X-ray diffraction (XRD), thermal analysis (TA), chemical analysis, and metallographic observation, Markovskii and Bezruk<sup>[7–11]</sup> made a comprehensive investigation on the phase equilibria and proposed a whole Mn-B phase diagram.<sup>[11]</sup> Six compounds were reported:  $\text{Mn}_4\text{B}$ ,  $\text{t-Mn}_2\text{B}$ ,  $\text{MnB}$ ,  $\text{Mn}_3\text{B}_4$ ,  $\text{MnB}_2$ , and  $\text{MnB}_4$ . Although all of these compounds were confirmed to exist, most of the equilibrium data reported by Markovskii and Bezruk<sup>[11]</sup> were updated by subsequent researchers.<sup>[12–23]</sup> In the following, the phase relationship in the Mn-B phase diagram is critically discussed.

In the range from  $\text{MnB}_2$  to pure B, Markovskii and Bezruk<sup>[11]</sup> reported two eutectic reactions:  $\text{L} \leftrightarrow \text{MnB}_2 + \text{MnB}_4$  at 1800 °C and  $\text{L} \leftrightarrow \text{MnB}_4 + \beta(\text{B})$  at 1880 °C. In addition,  $\text{MnB}_2$  was determined to have a large homogeneity range.  $\alpha(\text{B})$  was found to be stable at low temperature with a transformation to  $\beta(\text{B})$  at 1300 °C. However,  $\text{MnB}_4$  was confirmed to decompose into  $\text{MnB}_2$  and  $\beta(\text{B})$  at high temperature.<sup>[13,16,17,20,22,23]</sup> Andersson and Carlsson<sup>[17]</sup> reported the decomposition temperature to be between 1350 and 1400 °C. In addition,  $\text{MnB}_2$  decomposes to  $\text{Mn}_3\text{B}_4$  and  $\text{MnB}_4$ <sup>[13,14,16,17,20,22,23]</sup> at lower temperature. The decomposition temperature determined by Andersson and Carlsson<sup>[17]</sup> was between 1050 and 1100 °C, which was consistent with the result of Cely et al.<sup>[20]</sup> who reported the temperature to be between 1050 and 1150 °C. The homogeneity range of  $\text{MnB}_2$  was very narrow.<sup>[15,20]</sup> Moreover, the phases in the  $\text{MnB}_x$  ( $x > 2$ ) alloy after melting were  $\text{MnB}_2$  and  $\beta(\text{B})$ ,<sup>[16,17,20]</sup> which also indicates that an eutectic reaction  $\text{L} \leftrightarrow \text{MnB}_2 + \beta(\text{B})$  exists rather than  $\text{L} \leftrightarrow \text{MnB}_2 + \text{MnB}_4$  or  $\text{L} \leftrightarrow \text{MnB}_4 + \beta(\text{B})$ .

In the composition range from  $\text{MnB}$  to  $\text{MnB}_2$ , the invariant reactions reported by Markovskii and Bezruk<sup>[11]</sup> were  $\text{L} \leftrightarrow \text{MnB} + \text{Mn}_3\text{B}_4$  and  $\text{MnB}_2 + \text{L} \leftrightarrow \text{Mn}_3\text{B}_4$ . However, the experimental results of Hoyle<sup>[22]</sup> showed that the

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**Fig. 1** Calculated Mn-B phase diagram according to the parameters from Liao and Spear.<sup>[3]</sup> It should be mentioned that the Gibbs energies for pure elements are obtained from Dinsdale<sup>[5]</sup> since these data used by Liao and Spear<sup>[3]</sup> are not available. Such a treatment causes the calculated phase diagram differs with that from Liao and Spear.<sup>[3]</sup> However, we just address the miscibility gaps of the liquid phase, which are only relevant with the interaction parameters for liquid phase rather than the Gibbs energies for pure elements.

invariant reactions in this region are  $\text{MnB} + \text{L} \leftrightarrow \text{Mn}_3\text{B}_4$  and  $\text{L} \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_2$ . The experimental evidences from Hoyle<sup>[22]</sup> were adopted in the assessed Mn-B phase diagram<sup>[3,4]</sup> and were also accepted in the present modeling.

In the range from t-Mn<sub>2</sub>B to MnB, the only experimental investigation is due to Markovskii and Bezruk.<sup>[11]</sup> Therefore, the phase equilibria reported by them<sup>[11]</sup> were adopted in both the assessed phase diagram<sup>[3,4]</sup> and the present modeling.

In the composition range from pure Mn to t-Mn<sub>2</sub>B, Markovskii and Bezruk<sup>[11]</sup> reported two reactions:  $\text{L} \leftrightarrow \text{Mn}_4\text{B} + \alpha(\text{Mn})$  at 1180 °C and  $\text{L} + \text{t-Mn}_2\text{B} \leftrightarrow \text{Mn}_4\text{B}$  at 1285 °C. However,  $\alpha(\text{Mn})$  cannot be stable above 727 °C.<sup>[5]</sup> By means of chemical analysis, XRD, differential thermal analysis (DTA), and microscopy, Pradelli and Gianoglio<sup>[19]</sup> reinvestigated this part of the phase diagram and proposed an eutectic reaction ( $\text{L} \leftrightarrow \delta\text{Mn} + \text{t-Mn}_2\text{B}$  at 1211 °C) and a peritectoid reaction ( $\text{Mn}_4\text{B} \leftrightarrow \gamma\text{Mn} + \text{t-Mn}_2\text{B}$  at 1124 °C). In addition, Mn<sub>4</sub>B has a large homogeneity range from 20 to 33.3 at.% B. Later, Mn<sub>4</sub>B was refined to be o-Mn<sub>2</sub>B with orthorhombic structure.<sup>[21]</sup> The composition of o-Mn<sub>2</sub>B was actually determined to be o-Mn<sub>2</sub>B<sub>0.982</sub> when considering the occupancy of boron position.<sup>[21]</sup> Since the data reported by Pradelli and Gianoglio<sup>[19]</sup> were self-consistent, they were adopted in the assessed phase diagram<sup>[3,4]</sup> and the present modeling.

Andersson and Carlsson<sup>[17]</sup> reported the maximum solubility of Mn in  $\beta(\text{B})$  to be 4.2 at.% at high temperature.

**Table 1** List of symbols and crystallographic data of the phases in the Mn-B system

Symbol	Pearson symbol/Space group/ Prototype	Lattice parameters (Å)	Comments	References
Liquid (δMn)	...	...	...	...
	cI2	$a = 3.0806$	Pure Mn	Liao <sup>[3]</sup>
	Im3m			
	W			
(γMn)	cF4	$a = 3.8624$	Pure Mn	Liao <sup>[3]</sup>
	Fm3m			
	Cu			
(βMn)	cP20	$a = 6.3152$	Pure Mn	Liao <sup>[3]</sup>
	P4 <sub>1</sub> 32			
	βMn			
(αMn)	cI58	$a = 8.9139$	Pure Mn	Liao <sup>[3]</sup>
	I43m			
	αMn			
o-Mn <sub>2</sub> B <sub>0.982</sub>	oF48	$a = 14.54$	...	Liao <sup>[3]</sup>
	Fddd	$b = 7.291$		
	...	$c = 4.208$		
t-Mn <sub>2</sub> B	tI12	$a = 5.148$	...	Liao <sup>[3]</sup>
	I4/mcm	$c = 4.208$		
MnB	oP8	$a = 5.56$	...	Liao <sup>[3]</sup>
	Pnma	$b = 2.977$		
	FeB	$c = 4.145$		
Mn <sub>3</sub> B <sub>4</sub>	oI14	$a = 3.03$	...	Liao <sup>[3]</sup>
	Imm	$b = 12.86$		
	Ta <sub>3</sub> B <sub>4</sub>	$c = 2.96$		
MnB <sub>2</sub>	hP3	$a = 3.007$	...	Liao <sup>[3]</sup>
	P6/mmm	$c = 3.037$		
	AlB <sub>2</sub>			
MnB <sub>4</sub>	mC14	$a = 5.503$	...	Liao <sup>[3]</sup>
	C2/m	$b = 5.369$		
	Ni <sub>3</sub> Sn <sub>4</sub>	$\beta = 122.71$ °C $c = 2.949$		
MnB <sub>23</sub>	hR108	$a = 10.9907$	...	
	R-3m	$c = 23.9964$	βB solution	Liao <sup>[3]</sup>
	...			
βB	hR108	$a = 10.925$	Pure B	Liao <sup>[3]</sup>
	R3m	$b = 23.81$		
	...			
Mn <sub>4</sub> B <sup>a</sup>	oF40	$a = 14.53$	...	Liao <sup>[3]</sup>
	Fddd	$b = 7.293$		
	Mn <sub>4</sub> B	$c = 4.209$		
αMnB	oC8	...	...	Okamoto <sup>[4]</sup>
	Cmcm			
	CrB			
βMnB	oP8	...	...	Okamoto <sup>[4]</sup>
	Pnma			
	FeB			

<sup>a</sup>Mn<sub>4</sub>B and o-Mn<sub>2</sub>B<sub>0.982</sub> refer to the same phase

However, the temperature for the maximum solubility was not specified. As a result, the solubility of Mn in  $\beta$ (B) was considered to be 4.2 at.% at the eutectic temperature of the reaction  $L \leftrightarrow MnB_2 + \beta(B)$  in the present modeling.

Kiessling<sup>[12]</sup> reported that the solubility of B in (Mn) was negligible by XRD. However, Cely et al.<sup>[20]</sup> indicated that B could dissolve interstitially in (Mn), but did not give the solubility value. Since no significant solubility of B in (Mn) has been reported, the solubility was not considered in the present modeling.

For the MnB compound, Okamoto<sup>[4]</sup> reported that it has two structures.  $\alpha$ MnB is stable at low temperature and transforms to  $\beta$ MnB at 1050 °C. However, in the measurement of the isothermal sections at 800 °C of Tm-Mn-B system by Chaban et al.<sup>[24]</sup> and Sc-Mn-B system by Mikhaleenko et al.,<sup>[25]</sup>  $\alpha$ MnB and  $\beta$ MnB were observed at 800 °C by XRD, respectively. Consequently, there is not sufficient evidence to demonstrate the stability of  $\alpha$ MnB and  $\beta$ MnB for the moment. Thus, only one compound MnB was considered in the present modeling.

## 2.2 Thermodynamic Data

The partial enthalpies of Mn and B in liquid in the composition range from pure Mn to 52 at.% B at 1677 °C were measured by Esin et al.<sup>[26]</sup> using a high-temperature calorimeter. B with a low purity (98 wt.% B) was used in their study. Also, the wrong difference between the enthalpy of B at 0 °C and that at 1677 °C was used in their derivation for the partial enthalpy of B. With the same method as that employed by Esin et al.,<sup>[26]</sup> Vitusevich<sup>[6]</sup> measured the partial enthalpies of B and Mn in liquid at 1600 °C. The experimental data<sup>[6]</sup> were adopted by Vitusevich,<sup>[27-29]</sup> who critically evaluated the thermodynamic data of binary and ternary melts of the 3d transition metals (Cr, Mn, Fe, Co, and Ni) with B. Consequently, the experimental data from Vitusevich<sup>[6]</sup> were utilized in the present modeling. By solution calorimetry, Kleppa and Sato<sup>[30]</sup> determined the

enthalpies of formation of t-Mn<sub>2</sub>B, MnB, and MnB<sub>2</sub> at 1113 °C, which were considered in the modeling.

The experimental data used in the optimization are listed in Table 2. Only the data from No. 1 to 11 were considered to be reliable as discussed above. In addition, since only Markovskii and Bezruk<sup>[11]</sup> have studied the phase equilibria in the composition range from t-Mn<sub>2</sub>B to MnB, their reported phase equilibria data within this composition range were also used in the optimization. Other data from No. 13 to 16 were obtained from the assessed phase diagram due to Liao and Spear<sup>[3]</sup> since the temperatures for these reactions were not experimentally determined.

## 3. Thermodynamic Model

The Gibbs energy function  ${}^0G_i^\phi(T) = G_i^\phi(T) - H_i^{\text{SER}}$  for the pure element  $i$  ( $i = \text{Mn}, \text{B}$ ) is expressed as:

$${}^0G_i^\phi(T) = a + b \cdot T + c \cdot \ln T + d \cdot T^2 + e \cdot T^{-1} + f \cdot T^3 + g \cdot T^7 + h \cdot T^{-9}, \quad (\text{Eq } 1)$$

where  $H_i^{\text{SER}}$  is the molar enthalpy of the element  $i$  at 25 °C and 1 bar in its stable element reference (SER) state, and  $T$  is the absolute temperature. In the present modeling, the Gibbs energies as a function of temperature are taken from the SGTE compilation by Dinsdale.<sup>[5]</sup>

The Gibbs energy for  $\phi$  ( $\phi = \text{liquid}$  and  $\beta(\text{B})$ ) is described by the Redlich-Kister polynomial<sup>[31]</sup>:

$$G_m^\phi - H^{\text{SER}} = (1-x) \cdot {}^0G_{\text{Mn}}^\phi + x \cdot {}^0G_{\text{B}}^\phi + R \cdot T \cdot [(1-x) \cdot \ln(1-x) + x \cdot \ln x] + x \cdot (1-x) [a_0 + b_0 \cdot T + (1-2x) \cdot (a_1 + b_1 \cdot T) + \dots] \quad (\text{Eq } 2)$$

in which  $H^{\text{SER}}$  denotes  $(1-x) \cdot H_{\text{Mn}}^{\text{SER}} + x \cdot H_{\text{B}}^{\text{SER}}$ ,  $R$  is the gas constant, and  $x$  represents the mole fraction of B. The

**Table 2** List of the data used in the optimization

No	Experimental data	$T$ (°C)	Ref.
1	Liquidus between liquid and $\delta$ Mn	...	[19]
2	Liquidus between liquid and t-Mn <sub>2</sub> B	...	[19]
3	$L \leftrightarrow \delta\text{Mn} + t\text{-Mn}_2\text{B}$	1211	[19]
4	$\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \gamma\text{Mn} + t\text{-Mn}_2\text{B}$	1124	[19]
5	$\text{MnB} + L \leftrightarrow \text{Mn}_3\text{B}_4$	...	[22]
6	$L \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_2$	...	[22]
7	$L \leftrightarrow \text{MnB}_2 + \beta(\text{B})$	...	[16,17,20]
8	$\text{MnB}_2 \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_4$	1050-1100	[17]
9	$\text{MnB}_4 \leftrightarrow \text{MnB}_2 + \beta(\text{B})$	1350-1400	[17]
10	Partial enthalpy of mixing of liquid phase	1600	[27]
11	Enthalpy of formation of t-Mn <sub>2</sub> B, MnB, MnB <sub>4</sub> (-31.7, -35.8, -21.1 kJ/mol · atoms, respectively)	1113	[31]
12	Phase equilibria in t-Mn <sub>2</sub> B to MnB part	...	[11]
13	$\text{MnB} + L \leftrightarrow \text{Mn}_3\text{B}_4$	1827	[3]
14	$L \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_2$	1750	[3]
15	$L \leftrightarrow \text{MnB}_2$	1827	[3]
16	$L \leftrightarrow \text{MnB}_2 + \beta(\text{B})$	1730	[3]

## **Section I: Basic and Applied Research**

**Table 3** Optimized thermodynamic parameters in the Mn-B system

$L_{\text{B}, \text{Mn}}^{\text{L}} = -134141.3 - 15.7142 \cdot T$   
 $L_{\text{B}, \text{Mn}}^{\text{L}} = 32025.1 - 21.7659 \cdot T$   
 $L_{\text{B}, \text{Mn}}^{\text{L}} = 59907.4$   
 $L_{\text{B}, \text{Mn}}^{\text{L}} = -8723.6$   
 $\text{o-Mn}_2\text{B}_{0.982}: \text{Mn}_{0.670691}\text{B}_{0.329309}$   
 $G_{\text{Mn:B}}^{\text{o-Mn}_2\text{B}_{0.982}} = 0.670691 \cdot {}^0G_{\text{Mn}}^{\text{2Mn}} - 0.329309 \cdot {}^0G_{\text{B}}^{\text{BB}} = -31651.9 - 6.17 \cdot T$   
 $t\text{-Mn}_2\text{B}: \text{Mn}_{0.666667}\text{B}_{0.333333}$   
 $G_{\text{Mn:B}}^{t\text{-Mn}_2\text{B}} = 0.666667 \cdot {}^0G_{\text{Mn}}^{\text{2Mn}} - 0.333333 \cdot {}^0G_{\text{B}}^{\text{BB}} = -31700 - 6.4802 \cdot T$   
 $Mn\text{B}: \text{Mn}_{0.5}\text{B}_{0.5}$   
 $G_{\text{Mn:B}}^{\text{MnB}} = 0.5 \cdot {}^0G_{\text{Mn}}^{\text{2Mn}} - 0.5 \cdot {}^0G_{\text{B}}^{\text{BB}} = -35800 - 9.9995 \cdot T$   
 $Mn_3\text{B}_4: \text{Mn}_{0.428571}\text{B}_{0.571429}$   
 $G_{\text{Mn:B}}^{Mn_3B_4} = 0.428571 \cdot {}^0G_{\text{Mn}}^{\text{2Mn}} - 0.571429 \cdot {}^0G_{\text{B}}^{\text{BB}} = -38786.7 - 6.9904 \cdot T$   
 $Mn\text{B}_2: \text{Mn}_{0.333333}\text{B}_{0.666667}$   
 $G_{\text{Mn:B}}^{MnB_2} = 0.333333 \cdot {}^0G_{\text{Mn}}^{\text{2Mn}} - 0.666667 \cdot {}^0G_{\text{B}}^{\text{BB}} = -21100 - 12.556 \cdot T$   
 $Mn\text{B}_4: \text{Mn}_{0.2}\text{B}_{0.8}$   
 $G_{\text{Mn:B}}^{MnB_4} = 0.2 \cdot {}^0G_{\text{Mn}}^{\text{2Mn}} - 0.8 \cdot {}^0G_{\text{B}}^{\text{BB}} = -17744.3 - 4.5934 \cdot T$   
 $\beta\text{B}: (\text{B}, \text{Mn})_1$   
 $G_{\text{B}}^{\beta\text{B}} = GHSERB$   
 $G_{\text{Mn}}^{\beta\text{B}} = GHSERMn + 40$   
 $L_{\text{B}, \text{Mn}}^{\beta\text{B}} = -49500 - 21 \cdot T$   
 $\alpha\text{Mn}: (\text{Mn})_1(\text{B}, \text{Va})_1$   
 $G_{\text{Mn:Va}}^{\alpha\text{Mn}} = GHSERMn$   
 $G_{\text{Mn:B}}^{\alpha\text{Mn}} = GHSERMn + GHSERB$   
 $\beta\text{Mn}: (\text{Mn})_1(\text{B}, \text{Va})_1$   
 $G_{\text{Mn:Va}}^{\beta\text{Mn}} = GCUBMn$   
 $G_{\text{Mn:B}}^{\beta\text{Mn}} = GCUBMn + GHSERB$   
 $\gamma\text{Mn}: (\text{Mn})_1(\text{B}, \text{Va})_1$   
 $G_{\text{Mn:Va}}^{\gamma\text{Mn}} = GFCCMn$   
 $G_{\text{Mn:B}}^{\gamma\text{Mn}} = GFCCMn + GFCCB$   
 $\delta\text{Mn}: (\text{Mn})_1(\text{B}, \text{Va})_3$   
 $G_{\text{Mn:Va}}^{\delta\text{Mn}} = GBCCMn$   
 $G_{\text{Mn:B}}^{\delta\text{Mn}} = GBCCMn + 3 \cdot GBCCB$

interaction coefficients  $a_j$  and  $b_j$  ( $j = 0, 1, 2$ ) are to be evaluated in the optimization process.

The compounds  $\text{o-Mn}_2\text{B}_{0.982}$ ,  $\text{t-Mn}_2\text{B}$ ,  $\text{MnB}$ ,  $\text{Mn}_3\text{B}_4$ ,  $\text{MnB}_2$ , and  $\text{MnB}_4$  are modeled as stoichiometric phases. The Gibbs energy for each compound is given by the following expression:

$$G(\text{Mn}_{(1-x)}B_x) - (1-x) \cdot H_{\text{Mn}}^{\text{SER}} - x \cdot H_{\text{B}}^{\text{SER}} \\ = (1-x) \cdot {}^0G_{\text{Mn}}^{\alpha\text{Mn}} + x \cdot {}^0G_{\text{B}}^{\beta\text{B}} + A + B \cdot T, \quad (\text{Eq 3})$$

where  $A$  and  $B$  are to be optimized.

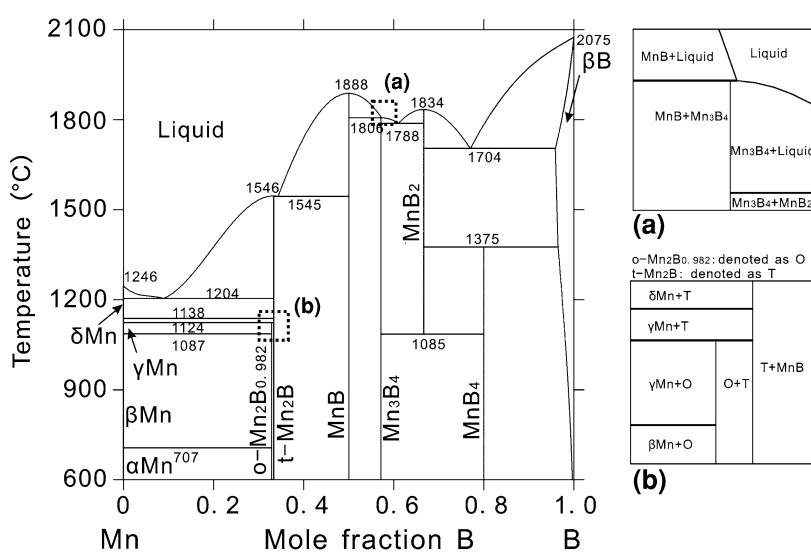
The Gibbs energy for  $\varphi$  ( $\varphi = \alpha\text{Mn}$ ,  $\beta\text{Mn}$ ,  $\gamma\text{Mn}$ , and  $\delta\text{Mn}$ ), is modeled by a two sublattice model ( $\text{Mn}_x(\text{B}, \text{Va})_y$ ). Its Gibbs energy expression is expressed as follows:

$$G_{\text{Mn:B,Va}}^{\varphi} = (1 - y''_{\text{B}}) \cdot G_{\text{Mn:Va}}^{\varphi} + y''_{\text{B}} \cdot G_{\text{Mn:B}}^{\varphi} + R \cdot T \cdot [x \cdot (1 - y''_{\text{B}}) \cdot \ln(1 - y''_{\text{B}}) + y \cdot y''_{\text{B}} \cdot \ln y''_{\text{B}}] + y''_{\text{B}} \cdot (1 - y''_{\text{B}}) \cdot [L_{\text{Mn:B,Va}}^0 + (1 - 2y''_{\text{B}}) \cdot L_{\text{Mn:B,Va}}^1 + \dots] + {}^{\text{mag}}G_{\text{m}}$$

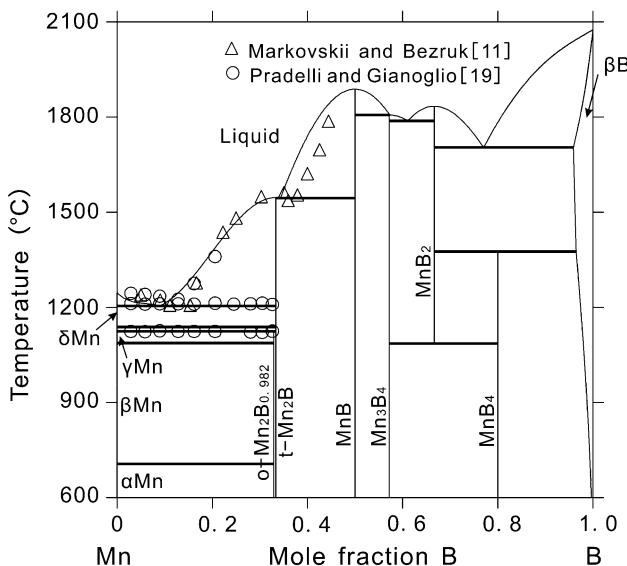
(Eq 4)

For  $\alpha$ Mn,  $\beta$ Mn, and  $\gamma$ Mn,  $x = y = 1$ , for  $\delta$ Mn,  $x = 1$ ,  $y = 3$ .  $y''_B$  represents the site fraction of B in the second sublattice.  $L_{\text{Mn}:B,\text{Va}}^j$  ( $j = 0, 1, 2$ ) denotes interaction between B and vacancy in the second sublattice. Since the solubility of B in (Mn) is not considered,  $L_{\text{Mn}:B,\text{Va}}^j$  is not used.

The  ${}^{\text{mag}}G_m$  term, which describes the magnetic contribution to the Gibbs energy, is given by the Hillert-Jarl-Inden model.<sup>[32,33]</sup> For  $\delta\text{Mn}$ ,  $\gamma\text{Mn}$ , and  $\alpha\text{Mn}$  phases, it is necessary to introduce  ${}^{\text{mag}}G_m$  term, while for  $\beta\text{Mn}$  phase, its  ${}^{\text{mag}}G_m$  term is equal to zero.



**Fig. 2** Calculated Mn-B phase diagram according to the present modeling



**Fig. 3** Calculated Mn-B phase diagram with the experimental data<sup>[11,19]</sup>

#### 4. Result and Discussion

The optimization began with the liquid phase. First, the parameters  $a_0$  and  $a_1$  for liquid phase could be optimized from the partial enthalpy of mixing.<sup>[6]</sup> Then, the optimization went to the Mn-MnB part and MnB-B part to adjust the parameters  $a$  and  $b$  for liquid phase and related compounds. As the solubility of Mn in  $\beta(B)$  is not appreciable,  $\beta(B)$  phase was considered later. Since o-Mn<sub>2</sub>B<sub>0.982</sub> and Mn<sub>3</sub>B<sub>4</sub> were not connected with liquid phase and the parameters for these two compounds would not influence the previous optimized equilibria, they were considered at last. The optimized parameters for this system are listed in Table 3. The calculated phase diagrams with and without experimental data are shown in Fig. 2 and 3, respectively. Table 4 lists the invariant equilibria along with the measured<sup>[11,17,19]</sup> and assessed<sup>[3,4]</sup> equilibria. For the optimization procedure and result, six points need to be addressed.

First, since there is no experimental evidence to indicate that any or all compounds except MnB<sub>2</sub> will decompose at low temperature, an assumption that these compounds except MnB<sub>2</sub> are stable at 0 °C was made in the optimization.

Second, as shown in Table 3,  ${}^0G_{\text{Mn}}^{\text{BB}} = \text{GHSERMn} + 40$  was introduced just to make  ${}^0G_{\text{Mn}}^{\text{BB}}$  not influence the Mn boundary, and the solubility of Mn in  $\beta(B)$  was adjusted by the interaction parameter  ${}^0L_{\text{B,Mn}}$ . Since there are no Gibbs energy functions for B with cub\_A13 or cbcc\_A12 structures, we use the approximation of  ${}^0G_{\text{Mn:B}}^{\text{aMn}} = \text{GHSERMn} + \text{GHSERB}$  and  ${}^0G_{\text{Mn:B}}^{\text{BMn}} = \text{GCUBMn} + \text{GHSERB}$ . Such a treatment does not cause any influence on the Mn rich region.

Third, the experimental δMn-L liquidus measured by Pradelli and Gianoglio<sup>[19]</sup> extends to 14.3 at.% B. However, the present modeling can only extend it to be 8.9 at.% B if

**Table 4** Calculated invariant equilibria along with the measured<sup>[11,17,19]</sup> and assessed<sup>[3,4]</sup> equilibria in the Mn-B system

Reaction	Reference	Composition (at.% B)		$T$ (°C)
		$L \leftrightarrow \delta\text{Mn} + t\text{-Mn}_2\text{B}$	$L \leftrightarrow t\text{-Mn}_2\text{B}$	
$L \leftrightarrow \delta\text{Mn} + t\text{-Mn}_2\text{B}$	This work	8.9	0	33.3
	Pradelli <sup>[19]</sup>	14.3	<1	33.3
	Liao <sup>[3]</sup>	15	0	33.3
	Okamoto <sup>[4]</sup>	14.3	<1	33.3
$L \leftrightarrow t\text{-Mn}_2\text{B}$	This work	33.3		1546
	Markovskii <sup>[11]</sup>	33.3		1580
	Liao <sup>[3]</sup>	33.3		1580
	Okamoto <sup>[4]</sup>	33.3		1580
$L \leftrightarrow t\text{-Mn}_2\text{B} + \text{MnB}$	This work	34.3	33.3	50
	Markovskii <sup>[11]</sup>	...	33.3	50
	Liao <sup>[3]</sup>	39	33.3	50
	Okamoto <sup>[4]</sup>	37	33.3	50
$L \leftrightarrow \text{MnB}$	This work		50	1888
	Markovskii <sup>[11]</sup>		50	1890
	Liao <sup>[3]</sup>		50	1890
	Okamoto <sup>[4]</sup>		50	1890
$L + \text{MnB} \leftrightarrow \text{Mn}_3\text{B}_4$	This work	57.3	50	57.1
	Liao <sup>[3]</sup>	57.3	50	57.1
	Okamoto <sup>[4]</sup>	...	50	57.1
$L \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_2$	This work	61.1	57.1	66.7
	Liao <sup>[3]</sup>	61.5	57.1	66.7
	Okamoto <sup>[4]</sup>	...	57.1	66.7
$L \leftrightarrow \text{MnB}_2$	This work		66.7	1834
	Liao <sup>[3]</sup>		66.7	1827
	Okamoto <sup>[4]</sup>		66.7	1990
$L \leftrightarrow \text{MnB}_2 + \beta\text{B}$	This work	77	66.7	95.9
	Liao <sup>[3]</sup>	80	66.7	100
	Okamoto <sup>[4]</sup>	80	66.7	>95.5
$\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \gamma\text{Mn} + t\text{-Mn}_2\text{B}$	This work	32.9	0	33.3
	Pradelli <sup>[19]</sup>	20-33	<1	33.3
	Liao <sup>[3]</sup>	25	0	33.3
$\text{MnB}_2 \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_4$	This work	66.7	57.1	80
	Andersson <sup>[17]</sup>	66.7	57.1	80
	Liao <sup>[3]</sup>	66.7	57.1	1100
$\text{MnB}_4 \leftrightarrow \text{MnB}_2 + \beta\text{B}$	This work	66.7	57.1	80
	Andersson <sup>[17]</sup>	80	66.7	...
	Liao <sup>[3]</sup>	80	66.7	100
	Okamoto <sup>[4]</sup>	80	66.7	1075
				1350-1400
				1375

the temperature for  $L \leftrightarrow \delta\text{Mn} + t\text{-Mn}_2\text{B}$  is kept around 1211 °C. The δMn-L liquidus determined by Pradelli and Gianoglio<sup>[19]</sup> is very flat, which has been mentioned by Okamoto.<sup>[4]</sup> We compare the δMn-L liquidus with those in other systems<sup>[34,35]</sup> involving B by introducing a parameter  $\xi$ :

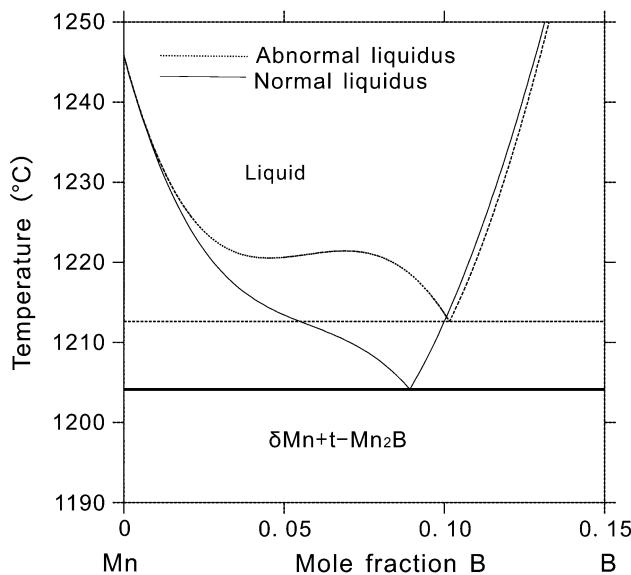
$$\xi = (T_m - T)/{}^L X_B \quad (\text{Eq } 5)$$

where  $T_m$  is the melting point of an element M which is close to Mn in the Periodic table.  $T$  is the temperature of the eutectic reaction  $L \leftrightarrow M + M_x\text{B}_y$ .  ${}^L X_B$  is the mole percent of B of liquid phase. The  $\xi$  for each system is listed in Table 5.

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**Table 5** Parameter  $\xi = (T_m - T)/{}^L X_B$  in the M-B system

System (M-B)	$T_m$ (°C)	Eutectic reaction	${}^L X_B$ (at.% B)	$T$ (°C)	$T_m - T$ (°C)	$\xi$
Mn-B	1246	$L \leftrightarrow \delta\text{Mn} + t\text{-Mn}_2\text{B}$	14.3	1211	35	2.4
Sc-B	1541	$L \leftrightarrow \beta\text{Sc} + \text{ScB}_2$	17	1277	264	15.5
Ti-B	1670	$L \leftrightarrow \beta\text{Ti} + \text{TiB}$	7	1540	130	18.6
V-B	1910	$L \leftrightarrow (\text{V}) + \text{V}_3\text{B}_2$	15	1735	175	11.7
Cr-B	1863	$L \leftrightarrow (\text{Cr}) + \text{Cr}_2\text{B}$	13.5	1630	233	17.3
Fe-B	1538	$L \leftrightarrow \gamma\text{Fe} + \text{Fe}_2\text{B}$	17	1174	364	21.4
Co-B	1495	$L \leftrightarrow \alpha\text{Co} + \text{Co}_3\text{B}$	18.5	1110	385	20.8
Ni-B	1455	$L \leftrightarrow (\text{Ni}) + \text{Ni}_3\text{B}$	17	1093	362	21.3
Cu-B	1085	$L \leftrightarrow (\text{Cu}) + \text{B}$	13.3	1013	72	5.4
Y-B	1522	$L \leftrightarrow \alpha\text{Y} + \text{YB}_2$	25.5	1290	232	9.1
Zr-B	1855	$L \leftrightarrow \beta\text{Zr} + \text{ZrB}_2$	13.7	1662	193	14.1
Nb-B	2469	$L \leftrightarrow (\text{Nb}) + \text{Nb}_3\text{B}_2$	14	1600	869	62.1
Mo-B	2623	$L \leftrightarrow (\text{Mo}) + \text{Mo}_2\text{B}$	23	2175	448	19.5



**Fig. 4** Abnormal and normal  $\delta\text{Mn}/\text{L}$  liquidus in the optimization

It can be clearly seen that  $\xi$  for the Mn-B system is significant smaller than that for any other system. This means that the Mn-rich liquidus in the Mn-B system is the flattest. In the optimization, the adjustment for the  $\delta\text{Mn}/\text{L}$  liquidus should be carried out very carefully. This flat liquidus tends to be not monotonic, as shown by the dashed line in Fig 4. Although the calculated liquidus from the previously parameters<sup>[3]</sup> can extend to 14.3% B, an abnormal liquid miscibility gap appears in the Mn-rich corner when using the Thermo-Calc S version or Pandat software. Since we keep the temperature for  $L \leftrightarrow \delta\text{Mn} + t\text{-Mn}_2\text{B}$  around 1211 °C and make the  $\delta\text{Mn}/\text{L}$  liquidus monotonic as B content increases, the calculated eutectic composition at 8.9 at.% B instead of 14.3 at.% B can be accepted. Because there are no experimental data of greater reliability for this system, no attempt to use more parameters

for the adjustment of this liquidus was not considered since using more parameters is not encouraged by the CALPHAD optimization spirit.

Fourth, the parameters  $A$  and  $B$  for  $\text{o-Mn}_2\text{B}_{0.982}$  were determined by the reaction  $\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \gamma\text{Mn} + t\text{-Mn}_2\text{B}$  at 1124 °C and the limitation condition  $\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \alpha\text{Mn} + t\text{-Mn}_2\text{B}$  and  $t\text{-Mn}_2\text{B} \leftrightarrow \text{o-Mn}_2\text{B}_{0.982} + \text{MnB}$  do not happen at 0 °C. They require that  $-6.4113 < B < -6.1605$  and  $A + B \cdot 1397.15 = -40272.2882$ . The derivation of the relation is discussed in the following.

For  $\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \gamma\text{Mn} + t\text{-Mn}_2\text{B}$ , we transform it to

$$2 \cdot \text{o-Mn}_2\text{B}_{0.982} = 0.072 \cdot \gamma\text{Mn} + 1.964 \cdot t\text{-Mn}_2\text{B} \quad (\text{Eq } 6)$$

If this reaction happens at  $T$  °C, we get

$$\begin{aligned} 2 \cdot G(\text{o-Mn}_2\text{B}_{0.982}) &= 0.072 \cdot G(\gamma\text{Mn}) \\ &\quad + 1.964 \cdot G(t\text{-Mn}_2\text{B}) \quad \text{at } T \text{ °C} \end{aligned} \quad (\text{Eq } 7)$$

When  $T = 1124$  °C,  $G(\gamma\text{Mn})$  and  $G(t\text{-Mn}_2\text{B})$  are obtained from Dinsdale<sup>[5]</sup> and previous modeling, respectively. Inserting these values into Eq 7, we get

$$A + B \cdot 1397.15 = -40272.2882 \quad (\text{Eq } 8)$$

for the reaction  $\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \gamma\text{Mn} + t\text{-Mn}_2\text{B}$  at 1124 °C.

Next, the limitation condition is considered. For  $\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \alpha\text{Mn} + t\text{-Mn}_2\text{B}$  does not happen at 0 °C, we get

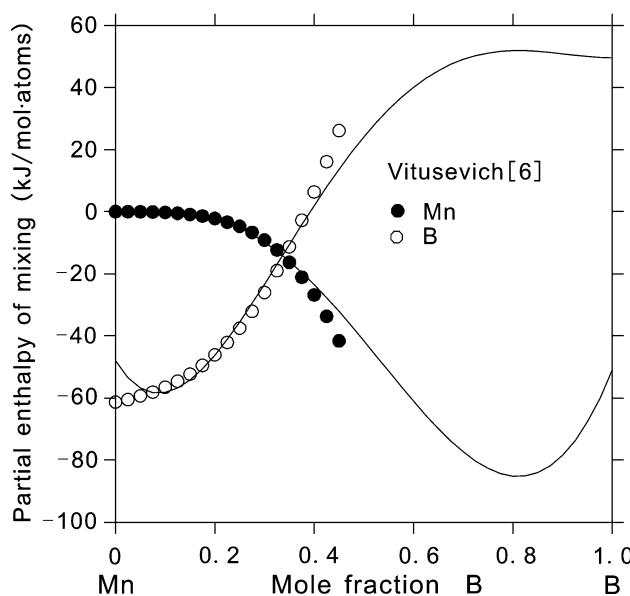
$$\begin{aligned} 2 \cdot G(\text{o-Mn}_2\text{B}_{0.982}) &< 0.072 \cdot G(\alpha\text{Mn}) + 1.964 \\ &\quad \cdot G(t\text{-Mn}_2\text{B}) \quad \text{at } 0 \text{ °C} \end{aligned} \quad (\text{Eq } 9)$$

Similar as the derivation discussed above, we get

$$A + B \cdot 273.15 < -33066.0172 \quad (\text{Eq } 10)$$

Similarly, for  $t\text{-Mn}_2\text{B} \leftrightarrow \text{o-Mn}_2\text{B}_{0.982} + \text{MnB}$  does not happen at 0 °C, we get

$$A + B \cdot 273.15 > -33347.8623. \quad (\text{Eq } 11)$$



**Fig. 5** Calculated partial enthalpies of mixing for Mn and B in liquid phase compared with the experimental data.<sup>[6]</sup> The reference states are liquid Mn and solid  $\beta$ B at 1600 °C

If we put Eq 8 into Eq 10 and 11, we get

$$-6.4113 < B < -6.1605 \quad (\text{Eq 12})$$

As a result, only if  $A + B \cdot 1397.15 = -40272.2882$  with  $-6.4113 < B < -6.1605$ ,  $\text{o-Mn}_2\text{B}_{0.982} \leftrightarrow \gamma\text{Mn} + \text{t-Mn}_2\text{B}$  at 1124 °C and stabilities of  $\text{o-Mn}_2\text{B}_{0.982}$  and  $\text{t-Mn}_2\text{B}$  at 0 °C can be satisfied.

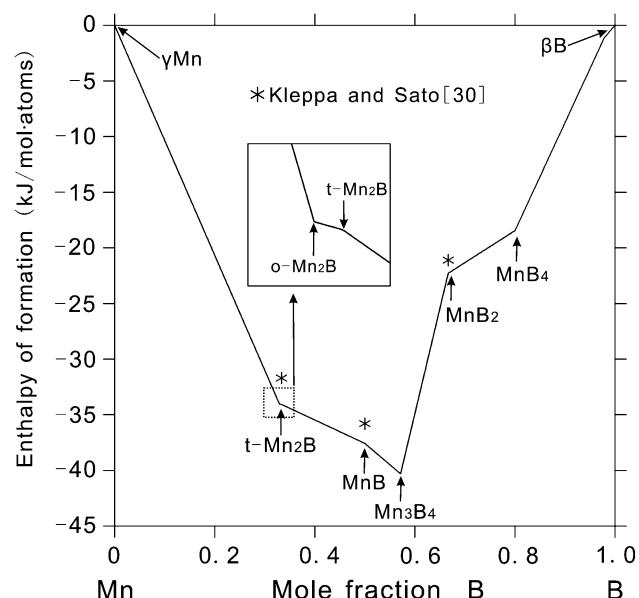
Since there are groups of  $A$  and  $B$  for  $\text{o-Mn}_2\text{B}_{0.982}$ , we arbitrarily chose one group:

$$B = -6.17 \quad A = -31651.9$$

However, the parameters  $A$  and  $B$  for  $\text{MnB}_2$  are fixed by  $\text{MnB}_2 \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_4$  and  $\text{MnB}_4 \leftrightarrow \text{MnB}_2 + \beta\text{B}$ . These two parameters should satisfy the limitation condition that  $\text{MnB}_4 \leftrightarrow \beta\text{B} + \text{Mn}_3\text{B}_4$  and  $\text{Mn}_3\text{B}_4 \leftrightarrow \text{MnB} + \text{MnB}_4$  do not happen at 0 °C. We attempted to make the  $\text{MnB}_2 \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_4$  and  $\text{MnB}_4 \leftrightarrow \text{MnB}_2 + \beta\text{B}$  to happen at 1075 and 1375 °C, respectively. However, they could not be satisfied completely at the same time because of the limitation condition. As a result, the  $\text{MnB}_4 \leftrightarrow \text{MnB}_2 + \beta\text{B}$  was adjusted to 1375 °C while  $\text{MnB}_2 \leftrightarrow \text{Mn}_3\text{B}_4 + \text{MnB}_4$  was at 1085 °C.

Fifth, as shown in Fig. 3, the calculated phase diagram in the composition range from  $\text{t-Mn}_2\text{B}$  and  $\text{MnB}$  is not consistent with the experimental data from Markovskii and Bezruk.<sup>[11]</sup> In the optimization, it seems that it is impossible to completely reproduce the phase diagram data in the composition ranges from (Mn) to  $\text{t-Mn}_2\text{B}$ <sup>[19]</sup> and from  $\text{t-Mn}_2\text{B}$  to  $\text{MnB}$ .<sup>[11]</sup> In the assessment, more weight was given to the data in the range from (Mn) to  $\text{t-Mn}_2\text{B}$ .<sup>[19]</sup>

At last, the calculated thermodynamic properties are in good agreements with the experimental data<sup>[6,30]</sup> as shown in Fig. 5 and 6. The present calculated phase diagram is in



**Fig. 6** Calculated enthalpy of formation compared with the experimental data.<sup>[30]</sup> The reference states are  $\gamma\text{Mn}$  and  $\beta\text{B}$  at 1113 °C

general agreement with the assessed phase diagram by Liao and Spear<sup>[3]</sup> as shown in Table 4. Although the temperatures for some invariant reactions show some differences from those in the assessed phase diagram by Okamoto,<sup>[4]</sup> all of calculated reaction types associated with the equilibria are the same as those assessed<sup>[3,4]</sup> or measured. Since some phase equilibria information used in the optimization was obtained from the assessed phase diagram by Liao and Spear,<sup>[3]</sup> additional experiments are needed.

## 5. Conclusion

- The Mn-B system is modeled based on critical literature review. A set of self-consistent parameters is obtained and the calculated results show general agreements with the reliable experimental data.
- The optimization procedure and results are critically discussed, particularly for the flat  $\delta\text{Mn}/\text{L}$  liquidus,  $\text{o-Mn}_2\text{B}_{0.982}$  and  $\text{MnB}_4$ . The  $\delta\text{Mn}/\text{L}$  liquidus is determined to be the flattest when compared with the Metal/L liquidus in other systems. Such a flat liquidus makes modeling difficult. The ranges of the thermodynamic parameters for  $\text{o-Mn}_2\text{B}_{0.982}$  are mathematically derived.
- More experimental work is needed to refine the present thermodynamic modeling.

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